technische universität
dortmund

CP Violation types

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Fakultät Physik

Physics beyond the Standard Model

Outline

Introduction to CP Violation

CP Violation in decay

CP Violation in interference

Y. Grossman and P. Tanedo, "Just a taste: Lectures on flavor physics," in Anticipating the Next Discoveries in Particle Physics, WORLD SCIENTIFIC, May 2018.

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- Typically measured through ratios of branching ratios:

$$A_{CP} \equiv \frac{\Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f})}{\Gamma(B \to f) + \Gamma(\bar{B} \to \bar{f})}$$

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$$\mathcal{L}_{Wqq} = \frac{g}{\sqrt{2}} \bar{u}_L i\gamma_\mu d_L W^\mu \to \frac{g}{\sqrt{2}} \bar{u}_L i\gamma_\mu \left(V_{uL} V^\dagger_{dL}\right) d_L W^\mu$$

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- The CKM matrix can now be identified:

$$V \equiv V_{uL}V^\dagger_{dL}$$

  - Unitary
  - 4 physical parameters: 3 mixing angles + 1 complex phase

Parameterisation of the CKM matrix

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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- Wolfenstein parameterisation:

$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right)$$

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$$(CP)\mathcal{L}_{\text{Yuk},u} = -\frac{v}{2\sqrt{2}}\left[\bar{u}(Y_u + Y_u^\dagger)^T u - \bar{u}(Y_u - Y_u^\dagger)^T \gamma^5 u\right] =$$

$$= -\frac{v}{2\sqrt{2}}\left[\bar{u}(Y_u^\star + Y_u^{\dagger\star})u + \bar{u}(Y_u^\star - Y_u^{\dagger\star})\gamma^5 u\right]$$

→ The Lagrangian is CP-invariant ***only*** when $Y = Y^*$.

Reference: *M. D. Schwartz, Quantum Field Theory and the Standard Model. Cambridge University Press, 3 2014.*

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→ CP is conserved ***only*** if $|p| = |q|$.

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$$|A_f| = |\bar{A}_{\bar{f}}| \quad \text{if} \quad a_1 = 0 \ \text{ or } \ a_2 = 0 \ \text{ or } \ \delta_1 = \delta_2 \ \text{ or } \ \phi_1 = \phi_2$$

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- **CP Violation in interference between decays with and without mixing**. It is coming from the interference between a decay $B \to f$ and a decay $B \to \bar{B} \to f$. This is characterised by $\mathrm{Im}\,\lambda_f \neq 0$, where

$$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

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- For first order in r we get

$$a_{CP} = r \sin\Delta\phi \sin\Delta\delta$$

$B \to K\pi$

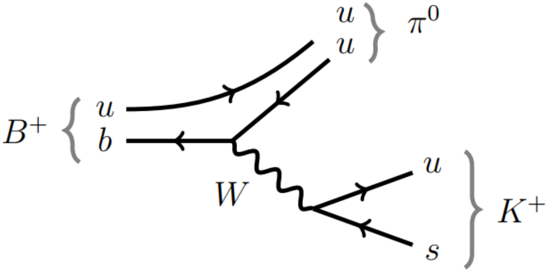


Figure: Tree level decay

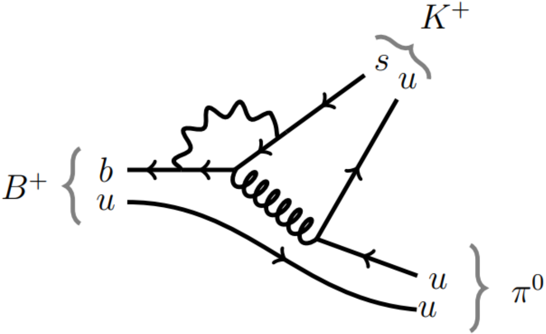
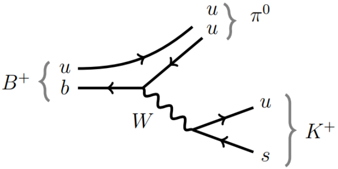


Figure: Penguin-mediated decay

$B \to K\pi$



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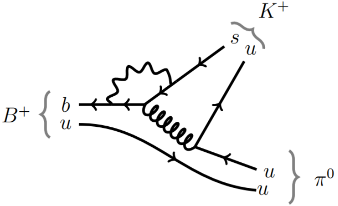

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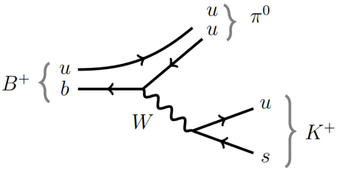

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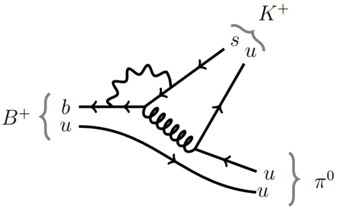

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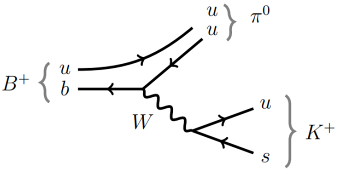

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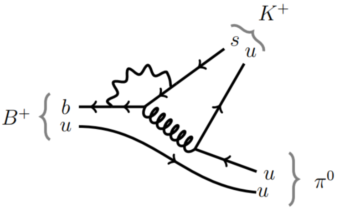

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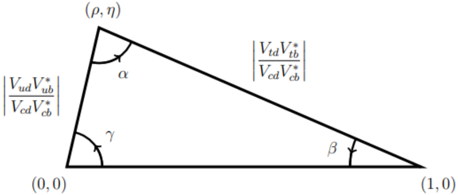
- Weak phase difference equal to γ due to V_{ub}.



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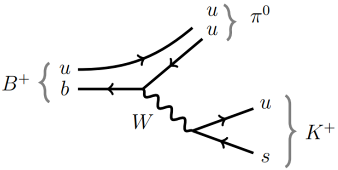


Figure: Tree level decay

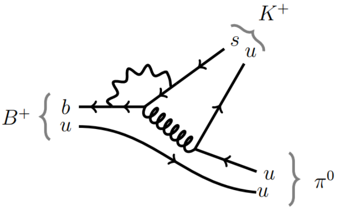
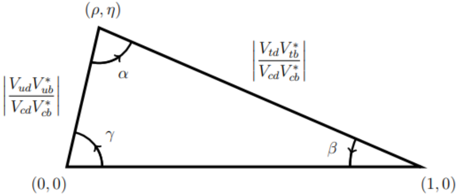


Figure: Penguin-mediated decay

- $\mathcal{M}_{\text{penguin}} \sim \frac{\alpha_w \alpha_s \lambda^2}{16\pi^2}$ $\mathcal{M}_{\text{tree}} \sim \alpha_w \lambda^4$
- Weak phase difference equal to γ due to V_{ub}.



- We know nothing about the strong phase.

$B \to DK$

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$$B^+ \to D^0 K^+ \qquad B^+ \to \bar{D}^0 K^+ \qquad B^+ \to D_{\text{CP}} K^+$$

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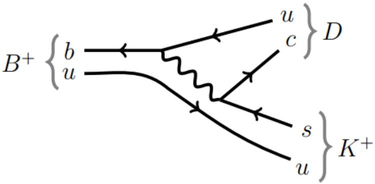

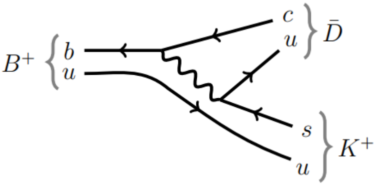

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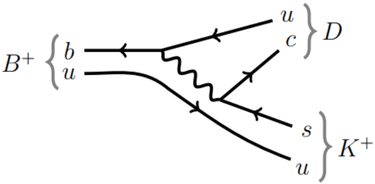

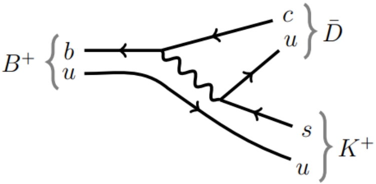

From $B^+ \to D^0 K^+$, the vertex $\bar{b}\bar{u}$ will make the angle γ of the Unitarity Triangle appear.

$B \to DK$

$$A_1^+ \equiv A(B^+ \to D^0 K^+) \quad A_2^+ \equiv A(B^+ \to \bar{D}^0 K^+) \quad A_{\mathrm{CP}}^+ \equiv A(B^+ \to D_{\mathrm{CP}} K^+)$$
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$$A_{\text{CP}}^{\pm} = \frac{A_1^{\pm} + A_2^{\pm}}{\sqrt{2}}$$

Defining $r \equiv \frac{|A_2^+|}{A}$,

$$A_1^+ = A \qquad A_2^+ = Are^{i(\gamma+\delta)}$$
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Computing the decay rates,

$$\Gamma[B^+ \to D_{\text{CP}} K^+] = |A_{\text{CP}}|^2 = \frac{A^2}{2}[1 + r^2 + 2r\cos(\gamma+\delta)]$$
$$\Gamma[B^- \to D_{\text{CP}} K^-] = |A_{\text{CP}}|^2 = \frac{A^2}{2}[1 + r^2 + 2r\cos(\delta-\gamma)]$$

CP Violation in interference

The observable

The observable

$$\left.\begin{array}{l} B \to f: \quad \Gamma[B^0(t) \to f] \\ \bar{B} \to f: \quad \Gamma[B^0(t) \to f] \end{array}\right\} \quad \mathcal{A}_{CP}(t) \equiv \frac{\Gamma[B^0(t) \to f] - \Gamma[\bar{B}^0(t) \to f]}{\Gamma[B^0(t) \to f] + \Gamma[\bar{B}^0(t) \to f]}$$

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$f = f_{\text{CP}} \implies CP\,|f\rangle = \eta_f |f\rangle$ where η is the CP parity

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$$A_f \equiv \langle f|T|B^0\rangle$$

$$\bar{A}_f \equiv \langle f|T|\bar{B}^0\rangle$$

$$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

$$\mathit{\Gamma}[B^0(t) \to f] = \left|A_f\right|^2 \left\{\left|g_+(t)\right|^2 + \left|\lambda_f\right|^2 \left|g_-(t)\right|^2 + 2\,\Re\left[\lambda_f g_+^*(t) g_-(t)\right]\right\}$$

$$\mathit{\Gamma}[\bar{B}^0(t) \to f] = \left|A_f\right|^2 \left|\frac{p}{q}\right|^2 \left\{\left|g_-(t)\right|^2 + \left|\lambda_f\right|^2 \left|g_+(t)\right|^2 + 2\,\Re\left[\lambda_f g_+(t) g_-(t)^*\right]\right\}$$

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$$\Delta\Gamma \simeq (2.7 \pm 0.4) \cdot 10^{-3}\text{ps}^{-1}$$

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For the CP conjugate,

$$\Gamma[\bar{B}^0(t) \to f] = \frac{\left|A_f\right|^2 e^{-\Gamma t}}{2} \left[1 + \left|\lambda_f\right|^2 - \left(1 - \left|\lambda_f\right|^2\right) \cos(\Delta m t) - 2\,\Im(\lambda_f) \sin(\Delta m t)\right]$$

[1] *Particle Data Group, P A Zyla et al. Review of Particle Physics. Progress of Theoretical and Experimental Physics, 2020(8):083C01, 08 2020.*

[2] *Johannes Albrecht et al. Lifetimes of b-hadrons and mixing of neutral B-mesons: theoretical and experimental status. Eur. Phys. J. ST, 233(2):359–390, 2024.*

Intermediate results

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Further assumptions

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Considering a single weak phase,

$$\left.\begin{aligned} A_f &= Ae^{i(\Phi_A+\delta)} \\ \bar{A}_f &= \eta_f A e^{i(-\Phi_A+\delta)} \end{aligned}\right\} \qquad \frac{\bar{A}_f}{A_f} = \frac{\eta_f A e^{i(-\Phi_A+\delta)}}{Ae^{i(\Phi_A+\delta)}} = \eta_f e^{-2i\Phi_A}$$

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We can conclude that λ_f is a **pure phase** ($|\lambda_f| = 1$).

The Holy Grail

Recall the results we have obtained until now:

- $\mathcal{A}(t) = a^{\text{dir}} \cos(\Delta m t) + a^{\text{int}} \sin(\Delta m t)$ with $a^{\text{dir}} = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}$ $\quad a^{\text{int}} = \frac{2\,\Im(\lambda_f)}{1+|\lambda_f|^2}$
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- $|\lambda_f| = 1$

We can conclude that the CP asymmetry is given by

$$\boxed{\mathcal{A}(t) = \Im(\lambda_f)\sin(\Delta m t)}$$

$B \to J/\Psi K_S$

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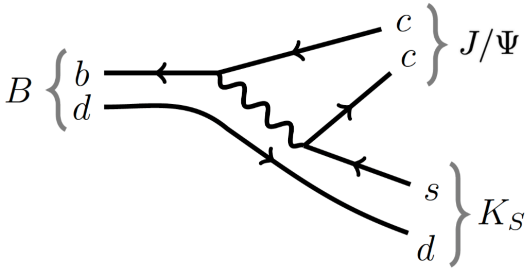

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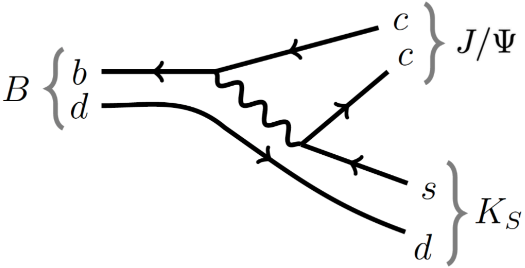

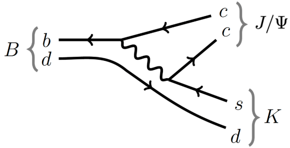

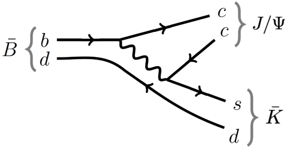

$$\left.\begin{array}{l} B^0 \longrightarrow K^0 \to K_S \\ B^0 \to \bar{B}^0 \to \bar{K}^0 \to K_S \end{array}\right\} \qquad \lambda_f = \frac{q_B}{p_B}\frac{\bar{A}_f}{A_f}$$

$B \to J/\Psi K_S$: Calculations

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We want to compute

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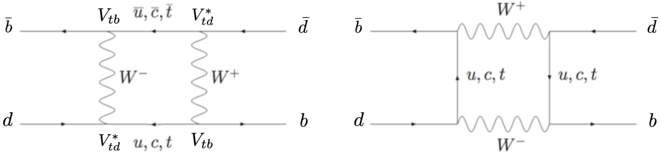

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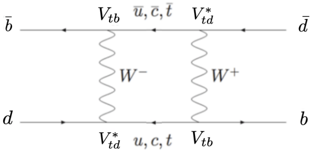

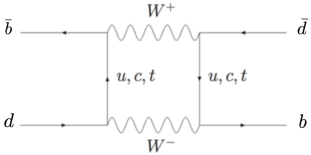

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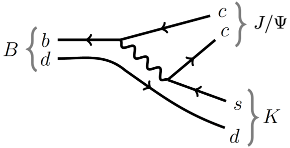
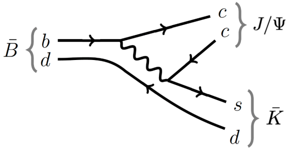
From the competing decays' diagrams,





$$\frac{\bar{A}_f}{A_f} \simeq \frac{p_K}{q_K} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \simeq \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}$$

$B \to J/\Psi K_S$: Results

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Merging the results,

$$\lambda_f \simeq -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} = -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{\cancel{V_{cs}} V_{cd}^*}{\cancel{V_{cs}^*} V_{cd}} \frac{V_{cb} \cancel{V_{cs}^*}}{V_{cb}^* \cancel{V_{cs}}} = -\frac{V_{cd}^* V_{cb}}{V_{td}^* V_{tb}} \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*}$$

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Using the definition of the β angle of the unitarity triangle,

$$\beta \equiv \text{arg}\left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right)$$

one gets

$$\lambda_f \simeq -\frac{-\left|\frac{V_{cd}^* V_{cb}}{V_{td}^* V_{tb}}\right| e^{-i\beta}}{-\left|\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right| e^{i\beta}} = -e^{-2i\beta} \quad \Longrightarrow \quad \Im(\lambda_f) \simeq \sin(2\beta)$$

$B \to J/\Psi K_S$: Results

Merging the results,

$$\lambda_f \simeq -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} = -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \frac{\cancel{V_{cs}} V_{cd}^*}{\cancel{V_{cs}^*} V_{cd}} \frac{V_{cb} \cancel{V_{cs}^*}}{V_{cb}^* \cancel{V_{cs}}} = -\frac{V_{cd}^* V_{cb}}{V_{td}^* V_{tb}} \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*}$$

Using the definition of the β angle of the unitarity triangle,

$$\beta \equiv \text{arg}\left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right)$$

one gets

$$\lambda_f \simeq -\frac{-\left|\frac{V_{cd}^* V_{cb}}{V_{td}^* V_{tb}}\right| e^{-i\beta}}{-\left|\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right| e^{i\beta}} = -e^{-2i\beta} \quad \Longrightarrow \quad \Im(\lambda_f) \simeq \sin(2\beta)$$

Finally,

$$\boxed{\mathcal{A}_{\text{CP}} \simeq \sin(2\beta)\sin(\Delta m t)}$$ From [3], $$\boxed{\sin(2\beta) = 0.699 \pm 0.017}$$

[3] *Particle Data Group, P A Zyla et al. Review of Particle Physics. Progress of Theoretical and Experimental Physics, 2020(8):083C01, 08 2020.*

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- Some CP-violating processes can be used to measure some fundamental parameters of the Standard Model: the angles of the Unitarity Triangle.